Exercise 1.1

Q.1 Identify each of the following as a rational or irrational numbers:

Solution:

Rational numbers

(i) 2.353535 (ii) $0.\overline{6}$ (ix) $\frac{15}{4}$ (x) $\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)$

Irrational numbers

(iii) 2.236067.. (iv) $\sqrt{7}$ (v) e (vi) π

(vii) $5+\sqrt{11}$ (viii) $\sqrt{3}+\sqrt{13}$

Q.2 Represent the following numbers on number line:

(i) $\sqrt{2}$

Solution:

$\sqrt{2}$ can be located on the real line by geometric construction. Mark a perpendicular line of $m\overline{AB}=1$ unit at A, where $m\overline{OA}=1$ unit, and we have a right-angle triangle OAB. By using Pythagoras theorem

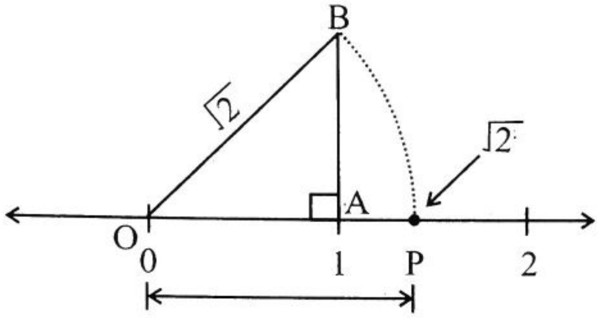
$$\left(m\overline{OB}\right)^2=\left(m\overline{OA}\right)^2+\left(m\overline{AB}\right)^2$$

$$\sqrt{\left(m\overline{OB}\right)^2}=\sqrt{\left(m\overline{OA}\right)^2+\left(m\overline{AB}\right)^2}$$

$$m\overline{OB}=\sqrt{(1)^2+(1)^2}$$

$$=\sqrt{1+1}$$

$$=\sqrt{2}$$



Taking O as centre, draw an arc of radius $m\,\overline{OB} = \sqrt{2}$ Which cut the number line at P. W e get point "P" representing $\sqrt{2}$ on the number line

So, $\left|\overline{OP}\right| = \sqrt{2}$

(ii) $\sqrt{3}$

Solution:

$\sqrt{3}$ can be located on the real line by geometric method. Mark a line of $m\overline{AB} = 1$ unit at A, With centre at A draw an arc of radius 2 units above the line. From point B draw a perpendicular line segment so that it cuts the arc at C. Join A to C. We have a right-angled ΔABC *in which* $m\overline{AB} = 1$ unit and $m\overline{AC} = 2$ units. By using Pythagoras theorem.

$$\left(m\overline{AC}\right)^2 = \left(m\overline{AB}\right)^2 + \left(m\overline{BC}\right)^2$$

$$(2)^2 = (1)^2 + \left(m\overline{BC}\right)^2$$

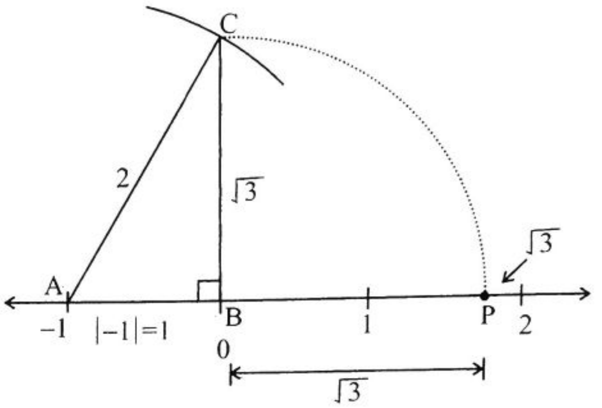
$$4 = 1 + \left(m\overline{BC}\right)^2$$

$$4 - 1 = \left(m\overline{BC}\right)^2$$

$$3 = \left(m\overline{BC}\right)^2$$

$$\sqrt{\left(m\overline{BC}\right)^2} = \sqrt{3}$$

$$m\overline{BC} = \sqrt{3}$$

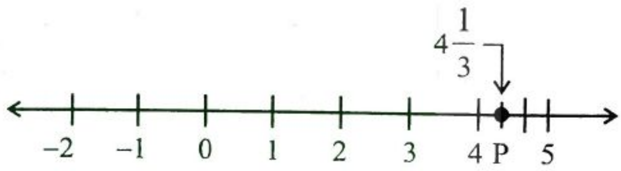


Now consider B is at 0. Taking B as centre, draw an arc of radius $m\,\overline{BC} = \sqrt{3}$, which cut the number line at P. We get point "P" representing $\sqrt{3}$ on the number line

So, $\left|\overline{BP}\right| = \sqrt{3}$

(iii) $4\frac{1}{3}$

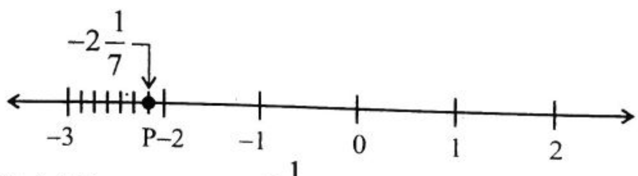
Solution:



Point P represents $4\frac{1}{3}$ on the number line.

(iv) $-2\frac{1}{7}$

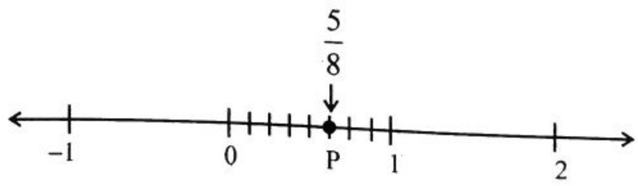
Solution:



Point P represents $-2\frac{1}{7}$ on the number line.

(v) $\frac{5}{8}$

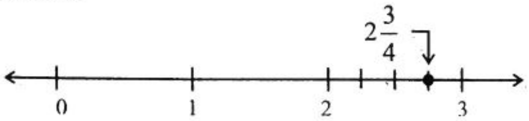
Solution:



Point P represents $\frac{5}{8}$ on the number line.

(vi) $2\frac{3}{4}$

Solution:



Point P represents $2\frac{3}{4}$ on the number line.

Q.3 Express the following as a rational number $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

(i) $0.\overline{4}$

Solution:

Let $x = 0.\overline{4}$

$x = 0.4444\ldots$ _________ (i)

Multiplying both sides by "10" , we get

$10x = 10\,(0.4444\ldots)$

$10x = 4.444\ldots$ _________ (ii)

Subtracting eq.(i) from (ii)

$10x - x = (4.444\ldots) - (0.4444\ldots)$

$9x = 4$

$x = \frac{4}{9}$

$\Rightarrow \; 0.\overline{4} = \frac{4}{9}$

(ii) $0.\overline{37}$

Solution:

Let $x = 0.\overline{37}$

$x = 0.37373737\ldots$ _________ (i)

Multiplying both sides by "100"

$100x = 100\;(0.37373737\ldots)$

$100x = 37.373737\ldots$ _________ (ii)

Subtracting eq.(i) from (ii)

$100x - x = (37.373737\ldots) - (0.37373737\ldots)$

$99x = 37$

$x = \frac{37}{99}$

$\Rightarrow \; 0.\overline{37} = \frac{37}{99}$

(iii) $0.\overline{21}$

Solution:

Let $x = 0.\overline{21}$

$x = 0.21212121\ldots$ _________ (i)

Multiplying both sides by "100"

$100x = 100\;(0.21212121\ldots)$

$100x = 21.212121\ldots$ _________ (ii)

Subtracting eq.(i) from (ii)

$100x - x = (21.212121\ldots) - (0..21212121\ldots)$

$99x = 21$

$x = \frac{21}{99} = \frac{7}{33}$

$\Rightarrow \; 0.\overline{21} = \frac{7}{33}$

Q.4 Name the property used in the following.

Solution:

Sr. No.		**Property Name**
(i)	$(a+4)+b = a+(4+b)$	Associative property w.r.t addition
(ii)	$\sqrt{2}+\sqrt{3} = \sqrt{3}+\sqrt{2}$	Commutative property w.r.t addition
(iii)	$x - x = 0$	Additive Inverse
(iv)	$a(b+c) = a\,b + a\,c$	Left distributive property of multiplication over addition.
(v)	$16 + 0 = 16$	Additive Identity
(vi)	$100 \times 1 = 100$	Multiplicative identity
(vii)	$4 \times (5 \times 8) = (4 \times 5) \times 8$	Associative property w.r.t multiplication
(viii)	$ab = ba$	Commutative property w.r.t multiplication.

5. Name the property used in the following:

Solution:

(i) $-3 < -1 \Rightarrow 0 < 2$

Additive property of inequality

(ii) If $a < b$ **then** $\frac{1}{a} > \frac{1}{b}$

Reciprocal property

(iii)If $a < b$ **then** $a+c < b+c$

Additive property of inequality

(iv)If $ac < bc$ **and** $c > 0$ **then** $a < b$

Cancellation property of inequality w.r.t multiplication.

(v) If $ac < bc$ **and** $c < 0$ **then** $a > b$

Cancellation property of inequality w.r.t multiplication.

(vi)Either $a > b$ **or** $a = b$ **or** $a < b$

Trichotomy property

6. Insert two rational numbers between

(i) $\frac{1}{3}$ **and** $\frac{1}{4}$

Solution:

Two rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$

Average of $\frac{1}{3}$ and $\frac{1}{4} = \left(\frac{1}{3}+\frac{1}{4}\right) \div 2$

$$= \left[\frac{4+3}{12}\right] \times \frac{1}{2}$$

$$= \frac{7}{12} \times \frac{1}{2} = \frac{7}{24}$$

Now we find average of $\frac{1}{3}$ and $\frac{7}{24}$

Average of $\frac{1}{3}$ and $\frac{7}{24} = \left(\frac{1}{3}+\frac{7}{24}\right) \div 2$

$$= \frac{8+7}{24} \times \frac{1}{2}$$

$$= \frac{15}{24} \times \frac{1}{2} = \frac{15}{48} = \frac{5}{16}$$

Thus $\frac{5}{16}$ and $\frac{7}{24}$ are two rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$.

(ii) 3 and 4

Solution:

Two rational numbers between 3 and 4.

Average of 3 and 4 $= \frac{3+4}{2} = \frac{7}{2}$

Average of $\frac{7}{2}$ and 4 $= \left(\frac{7}{2}+4\right) \div 2$

$$= \left(\frac{7+8}{2}\right) \div \frac{2}{1}$$

$$= \frac{15}{2} \times \frac{1}{2}$$

$$= \frac{15}{4}$$

Thus $\frac{7}{2}$ and $\frac{15}{4}$ are two rational numbers between 3 and 4.

(iii) $\frac{3}{5}$ **and** $\frac{4}{5}$

Solution:

Two rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$

Average of $\frac{3}{5}$ and $\frac{4}{5} = \left(\frac{3}{5}+\frac{4}{5}\right) \div 2$

$$= \left(\frac{3+4}{5}\right) \times \frac{1}{2}$$

$$= \frac{7}{5} \times \frac{1}{2} = \frac{7}{10}$$

Average of $\frac{7}{10}$ and $\frac{4}{5} = \left(\frac{7}{10}+\frac{4}{5}\right) \div 2$

$$= \left(\frac{7+8}{10}\right) \times \frac{1}{2}$$

$$= \frac{15}{10} \times \frac{1}{2}$$

$$= \frac{15}{20} = \frac{3}{4}$$

Thus $\frac{7}{10}$ and $\frac{3}{4}$ are two rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.